Fixed sequence integrated production and routing problems

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1 Introduction

The problem considered in this paper can be described as follows. A set of n jobs must be produced by a single machine and then delivered by a single vehicle.. Each job j requires a certain *processing time* p_j on a the machine, and must be delivered after its completion to the location j. We denote by t_{ij} the transportation time from destination i to destination j. We use M to denote the depot (manufacturer) and we assume that transportation times are symmetric and satisfy the triangle inequality. The set of jobs delivered during a single round trip constitutes a *batch*. The vehicle has a capacity c which is the maximum number of jobs it can load and hence deliver in a round trip. These models for coordinating production and delivery schedules have been largely analyzed and reviewed by Chen [2], who proposed a detailed classification scheme. In this paper the production sequence is *fixed* and the jobs must be delivered in the order in which they are released, hence a production sequence also specifies the sequence in which the customers have to be reached. Since the production sequence is given, the problem consists in determining a partition of all jobs into *batches*, and each batch will then be routed according to the manufacturing sequence.

The performance measures we consider in this paper (denoted f) is the *total delivery time*, i.e., $f = \sum_{j=1}^{n} D_j$, where D_j is the time at which the job $j = 1, \ldots, n$ is delivered at its destination.

Li et al [1] proved the NP-hardness of the general problem in which the sequence is not fixed and has to be decided. In this paper we show that the problem (denoted P) is already NP-hard when the sequence is fixed and we deal with the special case in which all distances t_{ij} are identical. For this special case, we show that the problem can be efficiently solved. Finally, we briefly enounce additional results. Detailed proofs are given in [4].

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2 Complexity

For our purposes, we introduce the Even-Odd Partition (EOP) problem. Given a set of n pairs of positive integers $(a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n)$, in which, for each i, $a_i > b_i$. Letting $K = \sum_{i=1}^{n}(a_i + b_i)$, is there a partition $(S, \bar{S})$ of the index set $I = \{1, 2, \ldots, n\}$ such that $\sum_{i \in S} a_i + \sum_{i \in \bar{S}} b_i = K/2$? EOP is proved NP-hard in the ordinary sense by Garey et al [3]. We will actually use a slightly modified version of the problem which remains equivalent to (EOP).

Modified Even-Odd Partition (MEOP). A set of n pairs of positive integers $(a_1, b_1), \ldots, (a_n, b_n)$ is given, in which, for each i, $a_i > b_i$. Letting $Q = \sum_{i=1}^{n}(a_i - b_i)$, is there a partition $(S, \bar{S})$ of the index set $I = \{1, 2, \ldots, n\}$ such that $\sum_{i \in S}(a_i - b_i) = Q/2$?

Theorem 1. *$P(\sum_{j=1}^{n} D_j)$ is NP-hard.*

Proof (Sketch). Given an instance of MEOP, we build an instance of P as follows. There are $3n + 3$ jobs. The processing times of the jobs are defined as follows: $p_1 = p_2 = p_3 = p_{3n+2} = p_{3n+3} = 0$, $p_{3n+1} = 4x_n + b_n + Q/2$ and for all $i = 1, \ldots, n-1$, $p_{3i+1} = 4x_i + b_i - 2$, $p_{3i+2} = 1$ and $p_{3i+3} = 1$. Where the x_i are defined as $x_i = (3a_i - 2b_i + 3(n-i)(a_i - b_i))/2$ for all $i = 1, \ldots, n$ and $x_{n+1} = 0$.

In the following, we refer to the set of jobs $(3(i-1)+1, 3(i-1)+2, 3i)$, $i = 1, \ldots, n$, as the *triple* T_i. For what concerns the travel times, we let:

$t_{M,3(i-1)+1} = t_{3(i-1)+1,M} = t_{M,3(i-1)+2} = t_{3(i-1)+2,M} = t_{M,3i} = t_{3i,M} = x_i \quad \forall i = 1, \ldots, n$

$t_{3(i-1)+1,3(i-1)+2} = a_i, t_{3(i-1)+2,3i} = b_i, t_{3i,3i+1} = x_i + x_{i+1} \quad \forall i = 1, \ldots, n$

$t_{M,3n+1} = t_{3n+1,M} = t_{M,3n+2} = t_{3n+2,M} = t_{M,3n+3} = t_{3n+1,3n+2} = t_{3n+2,3n+3} = 0$

Finally, vehicle capacity is $c = 2$. The problem consists in determining whether a solution exists such that the total delivery time does not exceed f^*.

$$f^* = \sum_{i=1}^{n}(3C_{3i} + 7x_i + b_i) + C_{3n+1} + C_{3n+2} + C_{3n+3} - Q/2. \tag{1}$$

Then we use the following Lemma.

Lemma 2. *If a schedule satisfying (1) exists, then there exists one satisfying the following property: for all $i = 1, \ldots, n+1$, jobs $3i$ and $3i+1$ are NOT in the same batch.*

Proof (omitted) □

Since the schedule satisfies Lemma 2 and $c = 2$, for each triple T_i, $i = 1, \ldots, n$, there are *exactly* two batches, and only two possibilities (see Figure 1), namely:

- *option A*: the first batch is $\{3(i-1)+1, 3(i-1)+2\}$ and the second is $\{3i\}$.
- *option B*: the first batch is $\{3(i-1)+1\}$ and the second is $\{3(i-1)+2, 3i\}$

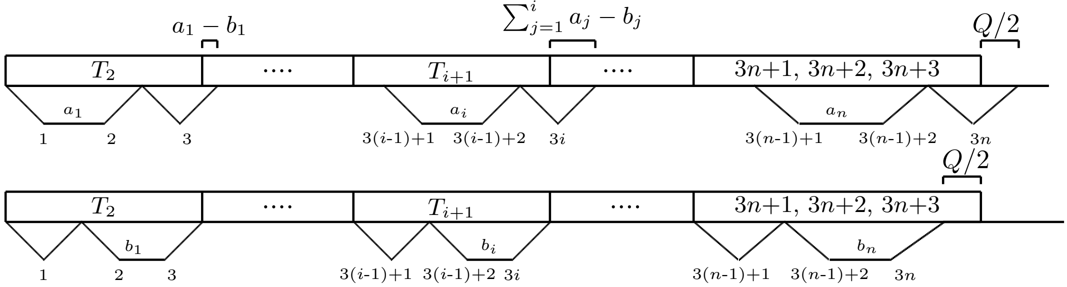

Figure 1: Round trips with options A or B.

From there, one can see that a schedule of value f^* exists if and only if EOP is a yes-instance. □

3 A special case: constant travel times

In this section we address the special case in which all travel times are identical. We start by analyzing some properties of the optimal solution then we give polynomial solution algorithm.

Clearly, every time the vehicle is back at the depot, it can either (i) restart immediately with a new batch consisting of jobs already completed, or (ii) it can wait for the completion of some jobs to be delivered. Suppose that a vehicle that departed at time t returns at the depot, and starts again at a certain time t'. Let $\mathcal{J}$ be the set of jobs released between t and t' (extremes included). The round trip starting at time t' is called *maximal* if either (i) the batch contains c jobs of $\mathcal{J}$, or (ii) it contains all jobs of $\mathcal{J}$. The next proposition gives a key feature of the optimal solutions.

Proposition 3. *There exists an optimal solution in which all round trips are maximal.*

Proof (omitted) □

Theorem 4. *Problem P with constant travel times can be solved in polynomial time.*

Proof (Sketch). Following Li et al [1], we call NSS (*non stop shipment*) a sequence of consecutive round trips during which the vehicle is never waiting at the depot, followed by a waiting time. We denote by $[i, j]$ an NSS starting at time C_i (hence, i is the last job of the first round trip of the NSS) and ending before C_j (when another NSS will start).

Starting from the proposition 3, we can construct in polynomial time our $[i, j]$ for all $i < j$. Note that, the last trip of $[i, j]$ contains a maximum number of jobs in order to finish the trip before C_j. We denote by $f(j)$ the following value of the optimal solution of the problem restricted to the first j jobs. Then, we have that recursive formula: $f(j) = \min_{\forall i<j}\{f(i) + F_{ij}\}$, where $F_{ij} = \sum_{k=i+1}^{j} D_k$ and $[i, j]$ an NSS. □

We also prove that the preemptive case in NP-hard. We study also the case where there is a fixed number of different locations. We propose pseudo-polynomial algorithms for the general case.

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